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Calculations of Electromagnetic Waves Propagation in Dielectric Medium in Three Dimensions

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Abstract

In this paper, we have applied the finite difference time domain (FDTD) method and the first order Mur boundary condition for calculating the electromagnetic (EM) by solving Maxwell's curl equations in three dimensions (3D) to study the EM propagation inside a dielectric medium in 3D and the calculations were performed using MATLAB. This study presented many numerical solutions to make a comparison between the calculations. The results clearly indicated that the slab can be utilized to control the direction of the EM propagation. The calculations demonstrated that the electromagnetic waves propagated and the maximum of the electromagnetic fields was concentrated inside the dielectric by the total internal reflections. The EM waves bilaterally propagated once the source is located in the middle of the slab. The EM waves can be produced 90 degree out of phase when the results are compared with each other. The calculations have verified that the distributions of the fields should be affected when the source setting changes, as the sources assigned to the electric component in the x -direction compared with y -direction. The final, two numerical solutions can be compared. The first one, the source was placed inside a dielectric which can be compared to the source being placed in free space. The results from the first calculation showed that the dielectric controls the propagation direction of the EM waves while the second solution demonstrates that the electromagnetic scattered by a dielectric.

Key words: finite difference time domain (FDTD) method, Maxwell's curl equations, absorbing boundary condition (ABC), three dimensions (3D) system.

حسابات انتشار الموجات الكهرومغناطيسية في وسط العازل في نظام ثلاثي الأبعاد

الصادق فرحات

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الملخص

تم في هذه الورقة محاكاة الموجات الكهرومغناطيسية بطريقة الفروق المحددة لحل معادلات ماكسويل في ثلاثة أبعاد وتطبيق شرط حدود مورمن درجة الاولى. تم حساب انتشار الموجات في مادة عازلة وحساب قيم المجال الكهربائي والمجال المغناطيسي في شبكة لعدة حالات مختلفة لغرض المقارنة بين النتائج حيث كل النتائج تشير الى تحديث قيم المجالات في كل حالة وتحكم في انتشار الموجات الكهرومغناطيسية في لوح العازل في ثلاثة أبعاد وأيضا تم حساب الموجات والمقارنة عندما تختلف في الطور بقيمة 90 درجة. تم اثبات في هذه الدراسة أنه من الممكن تغيير توزيع الموجي عند تغيير مصادر في اتجاه X مقارنة مع اتجاه Y وأيضا تم حساب و مقارنة حلول عديدة في هذه الورقة حيث الحل الأول وضع مصدر داخل العازل و مقارنته بوضع المصدر في الفضاء وقد أظهرت هذه الدراسة أن العازل يتحكم في اتجاه الانتشار بينما أظهر الحل الثاني أن الموجات الكهرومغناطيسية تشتت بواسطة اللوح العازل.

الكلمات المفتاحية: طريقة الفروق المحدودة، معادلات ماكسويل، شروط حدودية ممتصة، نظام ثلاثي الأبعاد

Introduction

Maxwell's curl equations can be used to describe many electromagnetic problems to determine the values of the electric and magnetic fields in a space by finding the analytical solutions. However, many problems are very difficult to solve without an approximation. To overcome this issue, the researchers apply many approaches by using the approximation methods. In this study, we will apply the finite difference time domain (FDTD) numerical method. The method applies in several fields for example of a propagation problem [1], a communication system [2], calculating a specific absorption rate (SAR), designing a waveguide [3], radio frequency coil [4] and plasma physics [5]. To obtain a very well

description of a problem, we require solving Maxwell's equations in three dimensions to observe the propagation direction in space. The propagation direction is very significant to consider, because of this area of research has a number of applications for example in a communication and microwave uses in medical [6].

The purpose of this work is to solve Maxwell's curl equations to study full EM wave propagation and the behavior of EM wave in a dielectric medium in the microwave range. The paper is organized as the follows: the three dimensions Maxwell's equations have described as the discrete equations and the absorbing boundary condition described in 3D system in method section, the next section has presented the calculations results that have displayed on different planes as the x - y plane and y - z plane as the images and the final section is the conclusion. We have calculated the EM waves interact with the shape filled with a dielectric of Teflon and the calculations can be performed by applying the FDTD method. In this work, the FDTD method can be applied to study the propagation of electromagnetic waves, which can be explained in detail in the next section.

Method

A number of methods are employed to find the solutions of Maxwell's equations in a problem space such as the FDTD method. This is time domain technique that will be applied to study the behavior of electromagnetic waves when the waves propagate inside a dielectric slab and scatter by the dielectric slab in 3D, the numerical method will apply such as the FDTD method. The first one applied the technique by Yee in the paper published in 1966 [7] and then the technique developed by many researchers to solve various complicated problems in one, two and three dimensions systems by solving Maxwell's equations.

1. Maxwell's curl equations:

To find any electromagnetic solution, Maxwell's curl equations must be solved which can be written when including a dielectric material in a space as the following [8]:

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\epsilon_r \epsilon_0} \nabla \times \mathbf{H} \quad (1)$$

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu_0} \nabla \times \mathbf{E} \quad (2)$$

Where: $E(i, j, k, t)$ is electric field, and $H(i, j, k, t)$ is magnetic field. The ε_0 and μ_0 are the permittivity and the permeability of free space, respectively. The ε_r is a relative permittivity.

2. Maxwell's equations in three dimensions (3D) system:

This section introduces Maxwell's equations in three dimensions (3D), any electromagnetic problem space could acquire very good explanation when solving the Maxwell's curl equations in 3D system. Therefore, it is essential to express Maxwell's equations in three dimensions (3D) system as [9]:

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon_r \varepsilon_0} \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) \quad (3)$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon_r \varepsilon_0} \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) \quad (4)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon_r \varepsilon_0} \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \quad (5)$$

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu_0} \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) \quad (6)$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0} \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) \quad (7)$$

$$\frac{\partial H_z}{\partial t} = -\frac{1}{\mu_0} \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) \quad (8)$$

3. The FDTD update equations for the electric and magnetic fields in 3D:

This part focuses on how to write Maxwell's equation in discrete forms to solve the problems in the 3D-FDTD which needs the full Yee algorithm that can be written in discrete forms by applying the central finite difference approximation with second order accuracy in a space derivative given by [10]:

$$\frac{\partial F^n(i, j, k)}{\partial x} = \frac{F^n(i+\frac{1}{2}, j, k) - F^n(i-\frac{1}{2}, j, k)}{\delta x} + O\delta^2 \quad (9)$$

Likewise, second order accuracy in time derivative provided by:

$$\frac{\partial F^n(i, j, k)}{\partial t} = \frac{F^{n+1/2}(i, j, k) - F^{n-1/2}(i, j, k)}{\delta t} + O\delta t^2 \quad (10)$$

The Eq. (9) and Eq. (10) substituted into Eq. (3) to Eq. (8) the spatial and temporal derivatives in order to represent the Eq. (3) to Eq. (8) in discrete forms of Maxwell's equations in the 3D to implement the equations in a computer program.

Therefore, we obtained the Eq. (11) to Eq. (16) and the forms are called the FDTD updating equations. There are six updating equations will be written. Three updating equations for the electric field components that expressed in three discretized equations are given by [10]:

$$E_x|_{i+\frac{1}{2},j,k}^{n+1} = E_x|_{i+\frac{1}{2},j,k}^n + \frac{1}{\varepsilon_r \varepsilon_0} \frac{\Delta t}{\delta} ((H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}} - H_z|_{i+\frac{1}{2},j-\frac{1}{2},k}^{n+\frac{1}{2}}) - (H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - H_y|_{i+\frac{1}{2},j,k-\frac{1}{2}}^{n+\frac{1}{2}})) \quad (11)$$

$$E_y|_{i,j+\frac{1}{2},k}^{n+1} = E_y|_{i,j+\frac{1}{2},k}^n + \frac{1}{\varepsilon_r \varepsilon_0} \frac{\Delta t}{\delta} ((H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}} - H_x|_{i,j+\frac{1}{2},k-\frac{1}{2}}^{n+\frac{1}{2}}) - (H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}} - H_z|_{i-\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}})) \quad (12)$$

$$E_z|_{i,j,k+\frac{1}{2}}^{n+1} = E_z|_{i,j,k+\frac{1}{2}}^n + \frac{1}{\varepsilon_r \varepsilon_0} \frac{\Delta t}{\delta} ((H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - H_y|_{i-\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}}) - (H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}} - H_x|_{i,j-\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}})) \quad (13)$$

Likewise, the FDTD update equations for the magnetic field, three discretized equations are provided by:

$$H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}} = H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n-\frac{1}{2}} + \frac{1}{\mu_0} \frac{\Delta t}{\delta} ((E_z|_{i,j,k+\frac{1}{2}}^n - E_z|_{i,j+1,k+\frac{1}{2}}^n) - (E_y|_{i,j+\frac{1}{2},k}^n - E_y|_{i,j+\frac{1}{2},k+1}^n)) \quad (14)$$

$$H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} = H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n-\frac{1}{2}} + \frac{1}{\mu_0} \frac{\Delta t}{\delta} ((E_z|_{i+1,j,k+\frac{1}{2}}^n - E_z|_{i,j,k+\frac{1}{2}}^n) - (E_x|_{i+\frac{1}{2},j,k+1}^n - E_x|_{i+1/2,j,k}^n)) \quad (15)$$

$$H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}} = H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n-\frac{1}{2}} + \frac{1}{\mu_0} \frac{\Delta t}{\delta} ((E_x|_{i+\frac{1}{2},j+1,k}^n - E_x|_{i+\frac{1}{2},j,k}^n) - (E_y|_{i+1,j+\frac{1}{2},k}^n - E_y|_{i,j+\frac{1}{2},k}^n)) \quad (16)$$

Where the superscript is indicated to the time instant and the indices as i, j and k represent the discretization in the x, y and z -directions, respectively.

Furthermore, from Eq. (11) can be seen that, every electric field component is surrounded by four adjacent magnetic field components. Similarly, every magnetic field component is surrounded by four adjacent electric field components. For example

of H_z component will be calculated by using four electric field components on different adjacent locations as shown in Eq. (16). Therefore, from figure 1 can be clearly seen that the arrangement of the electric and magnetic field components in each cell, this arrangement of the fields is called Yee's FDTD cell as the electric field components located on the edges of the cell and magnetic field components located on the faces of the cell as shown in figure 1. Once Yee's cells are added in a domain, this can generate a grid in 3D. From Eq. (11) to Eq. (16) can be normalized by applying this form: $\tilde{E} = \sqrt{\epsilon_0/\mu_0} E$ [11].

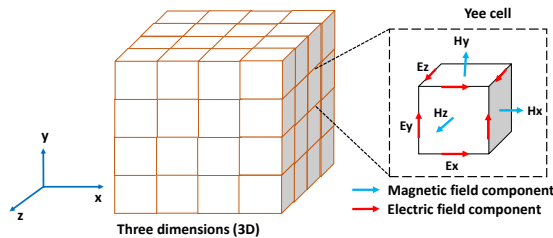


Figure 1: The positions of the electric and magnetic field components in Yee's FDTD cell in 3D [12].

4. Absorbing boundary condition (ABCs) in 3D-FDTD:

This section has explained the basic principles of the ABCs. Because of finding the solutions of differential equations usually require to an artificial boundary to limit the region of the calculation, as the number of the electric and magnetic fields should be finite. Therefore, we will explain how to implement the first order Mur's absorbing boundary condition (ABCs) that used to truncate the grid in three dimensions (3D) system at six sides as shown in figure 2. The ABCs are applied to obtain very good and accurate results, as the reflection will cause inaccurate results [13].

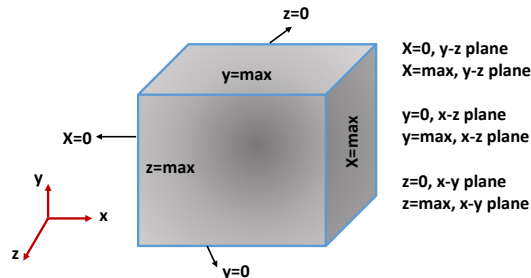


Figure 2: The six boundaries in three dimensions system in the x , y and z directions.

Therefore, the ABC must be included in the calculation to terminate the grid in three dimensions. The electromagnetic wave absorbs by the boundaries conditions once the waves reach the six sides. Therefore, the main advantage of applying ABC is to reduce a computational time and solve a problem without saving many arrays [10], the ABC can lead to reduce of saving unnecessary large amount of data. The calculation should be limited in the space and time. The six walls will behavior similar to a perfect electric conductor when there are no ABCs included in a domain. There are six surfaces must defined, from figure 2, the surface such as at $x=0$, the E_y and E_z must be terminated, based on Table 1, these components tangential to the surface. Consequently, two components (E_y and E_z) must be implemented as the ABCs, at $x=\max$, E_y and E_z are tangential to surface and must be updated by MUR ABC as shown in Eq. (17), the method can be reduced the electromagnetic waves reflections back into a computational domain.

Table 1 demonstrates the boundaries conditions of the first order MUR require updating in 3-D.

Boundaries	Applying MUR ABC
At $x=0$ and $x=\max$	Updating E_y and E_z because tangential to surface
At $y=0$ and $y=\max$	Updating E_x and E_z because tangential to surface
At $z=0$ and $z=\max$	Updating E_x and E_y because tangential to surface

In this case, at $x=0$, for the example of the E_z (ABC) is given by [14]:

$$E_z^{n+1}(0, j, k + \frac{1}{2}) = E_z^n(1, j, k + \frac{1}{2}) + (\frac{c\Delta t - \delta}{c\Delta t + \delta})(E_z^{n+1}(1, j, k + \frac{1}{2}) - E_z^n(0, j, k + \frac{1}{2})) \quad (17)$$

Where the δ is the space increment and Δt is the time increment. We have explained how to terminate the grid in computational domain at $x=0$. By utilizing the same methodology, the other field components of the absorbing boundaries conditions can be implemented, as the previous surface. The surface for the example of at the $y=0$ and $y=\max$, there are two field components must be implemented as the E_x and E_z because the field components tangential to this surface. Finally, the surface at $z=0$ and $z=\max$, there are two field components as the E_x and E_y must be included as

the ABC, because the components are tangential to the surface. Therefore, updating the absorbing boundary condition ABC based on utilize of Eq. (17), it requires its pervious value of the electric fields at the same node as well as its pervious value of the next node. The ABCs generate very good distribution as the reflected electromagnetic from the boundaries produce interference patterns. This will be seen in the results section.

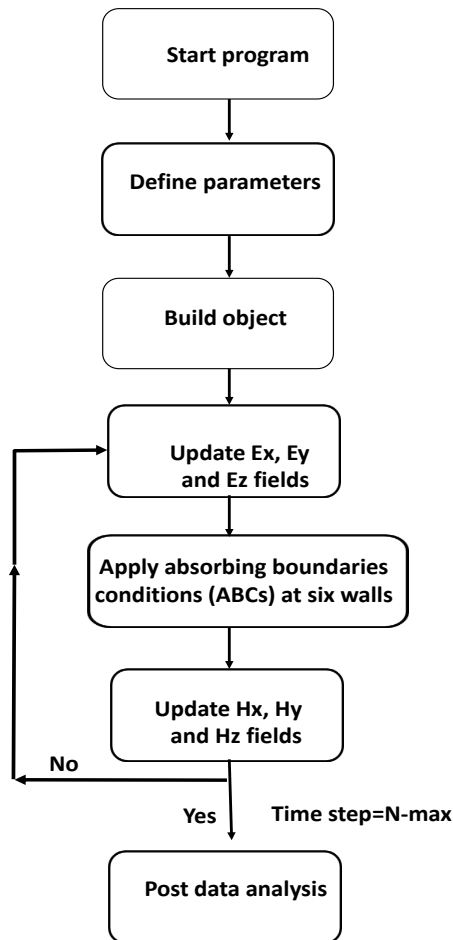


Figure 3: Flow chart of a program uses to generate the field components (E_x , E_y , E_z , H_x , H_y and H_z) in 3D by the FDTD technique.

Therefore, it is now quite clear that the ABCs should be updated every time step until the end of the iterations as seen in the flow chart shown in figure 3. It can be evaluated the performance of ABCs when implementing the ABCs in the program to truncate the six sides in the grid to calculate unbounded surrounding region. This

can be done by calculating the region of interest without an absorbing boundary compares with boundary condition to differentiate between the achieved results. In earlier sections, we have explained how to calculate the EM waves in three dimensions (3D) and the methodology to generate the electric and magnetic field components within a computational domain when a dielectric slab is included in the center of a space to consider the propagation of electromagnetic waves in three directions and the grid terminated by the ABCs.

Results and discussion

The results will demonstrate the propagation directions of the fields in three dimensions system; the programs were implemented in MATLAB to calculate electromagnetic waves. We have constructed the shape filled with dielectric placed in the free space. The aim of constructing the shape is to show how to electromagnetic waves propagate through the slab shaped dielectric and the structure is filled with a relative permittivity of Teflon ($\epsilon_r=2$) as shown in figure 4. We have displayed the structure on two different observations planes for the examples of the x - y plane and y - z plane as shown in figure 4.

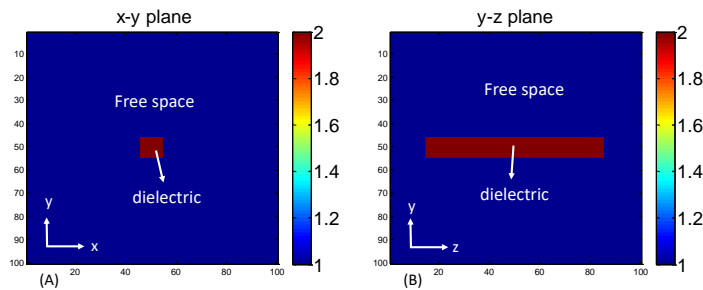


Figure 4: (A) cross section of a dielectric slab in the x - y plane and (B) in the y - z plane.

It can be noted that the computational domain consists of 100 by 100 by 100 cells in the x , y , and z directions, respectively. The field components will produce based on the use of the equations (11) to (16) to write a program using MATLAB as explained in the 3D-FDTD method in section 3. Two kinds of sources apply to excite a space such as the hard and soft sources [15] and many types of waveforms can use for examples a Gaussian pulse, modulated Gaussian pulse and a sinusoidal wave [16]. The hard point source

set as a sinusoidal wave and operated at 10 GHz in the microwave range.

Furthermore, the output of the results can be showed on different observation planes such as in the x - y plane and y - z plane to describe the behavior of electromagnetic waves. We can do a comparison between the calculations when exciting a space with different sources such as setting in the x -direction to make a comparison with y -direction. The second example demonstrates how to compare with pervious calculation when the domain excited 90 degree out of phase. Moreover, the aim of the third calculation is that considering the electromagnetic wave propagation when the source of excitation placed in middle of the dielectric slab. All snapshots obtained and demonstrated in this section computed at 400 time step in order to reach a steady state and obtain good coverage. We have calculated the system without any absorbing boundary condition to generate the electromagnetic wave in a domain. The result of the calculation illustrated in figure 5 can be clearly indicated that the electric and magnetic field components reached the six boundaries then the waves reflected back after that the incident fields combined with reflected fields. This calculation demonstrates the propagation of waves and their reflections and the result indicated that the distribution is not uniform. This is due to fact that the six boundaries acted similar to the perfect electric conductor.

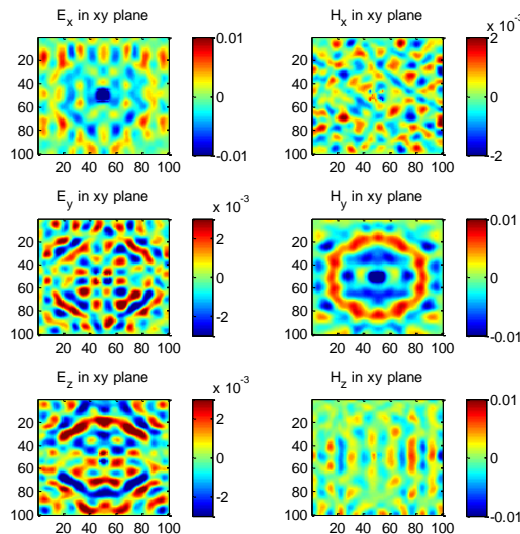


Figure 5: Source inserted in a dielectric slab and assigned in x -direction as well as there are no ABCs implemented at the boundaries.

On the other hand, the absorbing boundary condition can overcome this problem by implementing ABCs in six sides. By using the ABCs on each electric node on the six boundaries, the result of calculation demonstrated in figure 6 described that the circular patterns of the fields appeared in the results. Therefore, the waves appeared to propagate to infinite in a space which is called the system as an open computational domain. As results, the ABCs given in Eq. (17) acted very well to absorb the waves at the six sides.

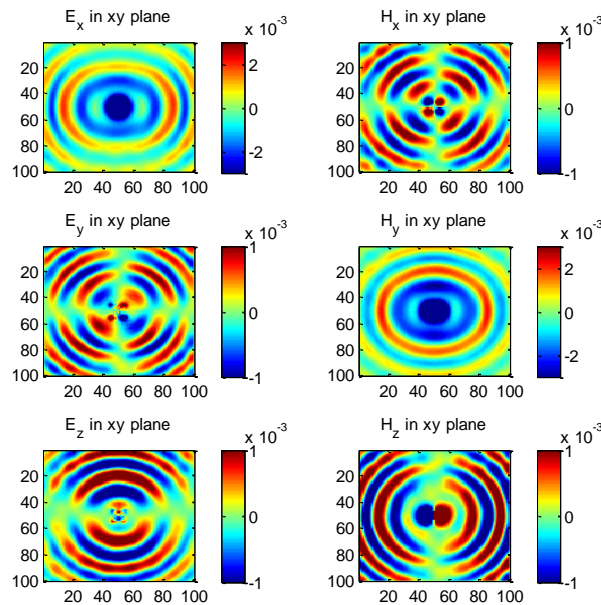


Figure 6: Source assigned in the x -direction and the ABCs at six sides.

In this research, we excited a domain by using many different ways such as the source assigned to the electric field in x -direction and in y -direction as shown in figure 6 compared with figure 7, respectively. The examples should demonstrate the different in the calculations. In addition, it can be excited a domain with line source, this would be applied instead of a point source as shown in figure 8. This example a space can be excited using a line source oriented in the x -direction to produce the waves as shown in figure 8, the distributions changed when compared with the example of a point source that generated in figure 6.

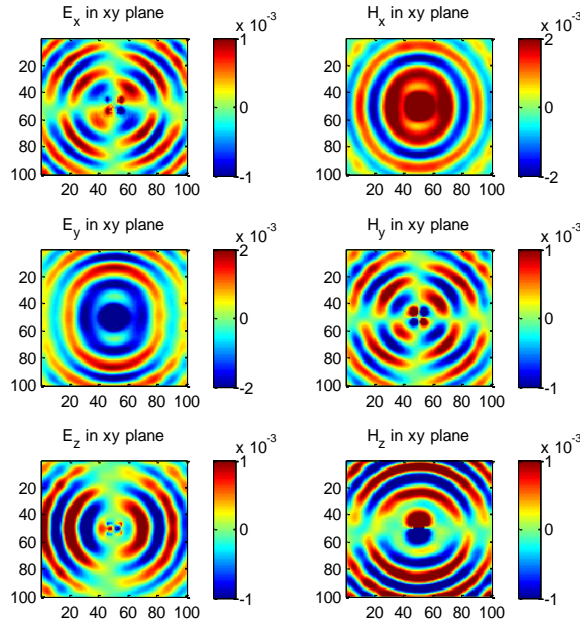


Figure 7: Source assigned in the y -direction.

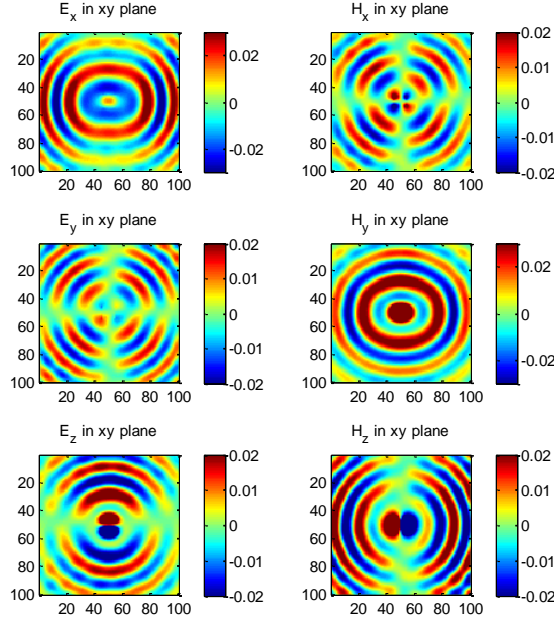


Figure 8: line Source assigned in the x -direction.

Furthermore, the computational domain can be excited out of phase 90 degrees as shown in the calculation results in figure 9. We found

that the distributions of the waves varied when generated the signals 90 degrees out of phase, as shown in the snapshots in figure 9 compared with figure 6, and there are two different snapshots appeared when excited out of phase.

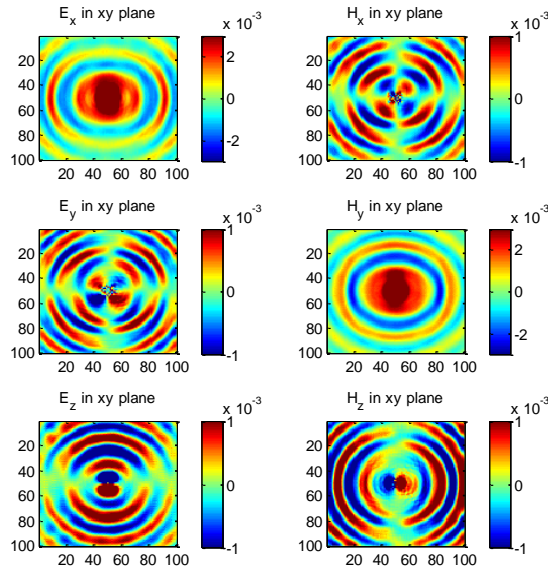


Figure 9: Source assigned in the x -direction and 90 degree out of phase.

It can be demonstrated that the signals propagated through a dielectric medium placed along- z direction as shown in figures 10, 11.

The results in the y - z planes appeared that when compared Figure 10 with figure 11, the distributions affected and changed when the domain excited by the source assigned in the x -direction compared with y -direction, respectively. It is obvious that the patterns flipped and the propagation direction controlled in the dielectric slab, and the distributions appeared very homogenous. It can be clearly observed that the propagation of electromagnetic concentrated inside the dielectric slab as many reflections generated in the slab to keep the waves propagation in a one direction in the medium, this is total internal reflections appeared in the calculations.

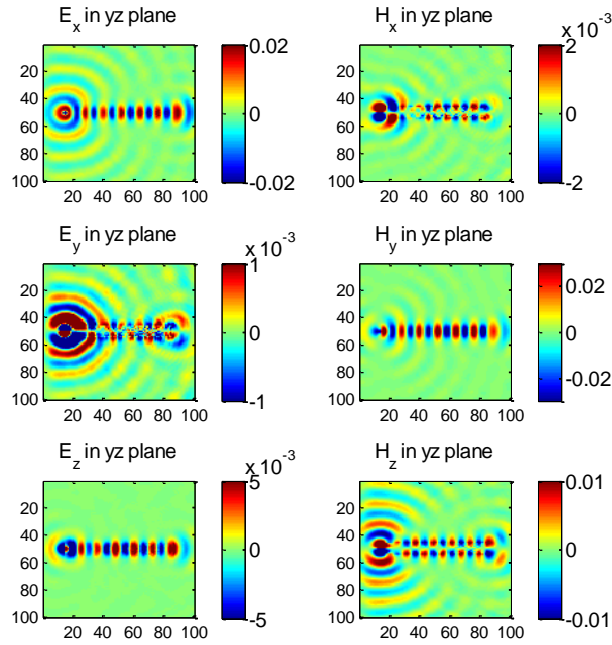


Figure 10: Source assigned in the x-direction and snapshots taken in the y-z plane.

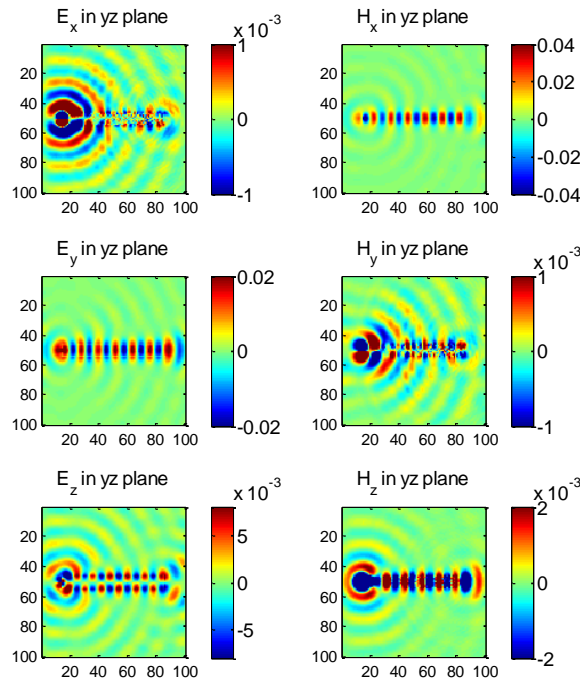


Figure 11: Source assigned in the y-direction and snapshots taken in the y-z plane.

As a result, the slab guided the waves to propagate at a specific direction. By excited the domain with the line source as shown in figure 12, uniformity of the fields improved and obtained higher intensities compared with calculation when a point source used as shown in figure 10. The signals appeared to propagate and concentrate in the dielectric slab.

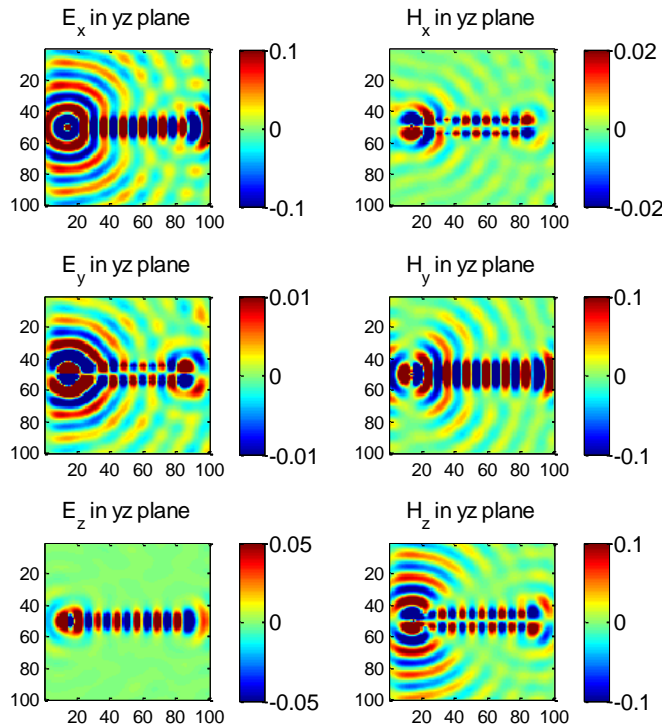


Figure 12: Line source assigned in the x -direction.

The previous calculations, the distributions are normalized and the source placed inside a dielectric slab. In the following calculation can be proved that a dielectric slab can be utilized to guide the EM in two directions once the source of exaction placed in the middle of a dielectric slab. The aim of this calculation is to generate the EM waves bilaterally as observed in figure 13, the signals guided in the slab and propagated bilaterally in the positive and negative directions in both sides with same distribution.

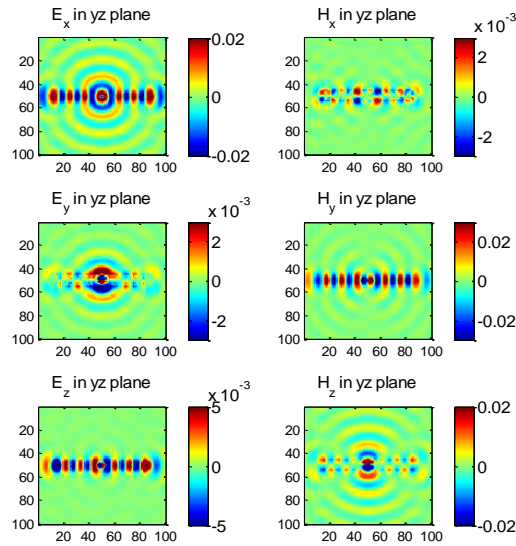


Figure 13: Source assigned in the x -direction and placed in the middle of dielectric slab.

From figure 14, the results indicated that the electromagnetic distribution changed when the source placed in a free space because the EM scattered by the slab. This calculation can be compared with prior calculations in the same planes, the previous calculation demonstrated that the EM concentrated in the slab while the last calculation the EM scattered by a dielectric. This calculation can confirm that the propagation direction depends on the source locations either in the free space or a dielectric.

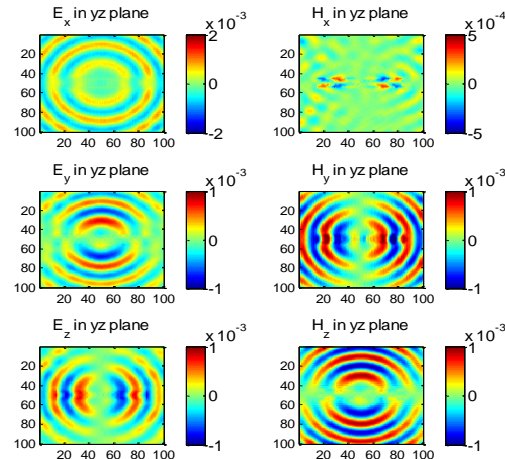


Figure 14: Source assigned in the x -direction and placed in free space.

Conclusion

We have explained the finite difference time domain (FDTD) technique after that the first order MUR absorbing boundary condition ABC in 3D. We have explained the calculations of full electromagnetic waves when the EM waves propagated inside a slab shaped dielectric that constructed in three dimensions. The results of calculations demonstrated that the FDTD method is very powerful to compute EM wave and the method can be utilized to describe how the EM controls the propagation direction inside the medium. The calculations results confirmed that the 3D Maxwell's equations as the discrete forms acted very well approach to solve the complicated geometry in the 3D-FDTD once the slab shaped dielectric constructed in three directions in a domain.

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